GASDYNAMIC REGIMES OF EVAPORATION AND
EXPLOSION OF A WATER AEROSOL DROPLET

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The heating, evaporation, and explosion regimes for a homogeneous water aerosol droplet are studied. Numerous theoretical and experimental results are analyzed and compared.

Several generalizing studies devoted to evaporation and explosion regimes for an aerosol droplet, heated by intense radiation, are known (see, for example, [1]), each of which complement the others in some respect because of the complexity of the physical process. The description of heating an individual droplet involves the equations of mass, momentum, and energy conservation inside the droplet, the balance equations for the above-mentioned quantities on the droplet surface, the relations for the jump of temperature, pressure, and density of the vapor in a thin Knudsen layer separating the liquid and gas phases, the conservation equations in an outer vapor-air mixture, and the equations of state for the water and the gas. The comparative analysis performed in the present study clearly testifies in favor of the hydrodynamic approach [2].

We will restrict our consideration to fairly small optically homogeneous droplets (with the initial radius $r_0 < \lambda$, α^{-1}, where λ is the radiation wavelength and α is the coefficient of radiation absorption by the water). A rate of the heat release inside the droplet is characterized by the quantity $q = \alpha_0(\rho_{cr}/\rho_w)^2 I_0$, where ρ_w and ρ_{cr} are the initial and critical densities of the water. Let us nondimensionalize the equations, taking characteristic quantities outside the droplet to be the initial temperature of the air T_0, the initial pressure in the air ρ_∞, the characteristic density ρ_v (of the vapor at $T_b = 373$ K or the initial density of the air), and the critical parameters of the water ρ_{cr}, p_{cr}, T_{cr}, and h_{cr} inside the droplet. On the assumption that the external medium is continuous, the Knudsen number is small, i.e., $Kn = \ell/r_0 << 1$ (ℓ is the free length of gas molecules). Using the familiar estimation $Re = PrPe = Pr_D Pe_D \sim M/Kn$ or $\eta_0 \sim \rho_v cl$, $k_0 \sim \rho_v C_p cl$, and $D_0 \sim cl$ (here η_0, k_0, and D_0 are the characteristic coefficients of viscosity, thermal conductivity, and diffusion for the mixture; C_p is the specific heat; c is the speed of sound in the mixture; $Re = \rho_v u_v r_0/\eta_0$, $Pe = \rho_v u_v C_p r_0/k_0$, $Pr = k_0/C_p\eta_0$, and $M = u_v/c$ are the Reynolds, Peclet, Prandtl, and Mach numbers; u_v is the characteristic velocity of the vapor; $Pe_D = u_v r_0/D_0$ and $Pr_D = D_0/\rho_v\eta_0$ are the diffusional Peclet and Prandtl numbers; for gases Pr, $Pr_D \sim 1$) we may distinguish four evaporation regimes in ascending order of the M number [3]: 1) diffusional, Pe_D, $Pe << 1$; $M << Kn$; 2) diffusional-convective, Pe_D, $Pe \sim 1$; $M \sim Kn << 1$; 3) subsonic, Pe_D, $Pe >> 1$; $M < 1$; and 4) sonic, $M_\ell \equiv 1$ (at the outer boundary of the Knudsen layer).

Solutions of the equations outside the droplet give the following expressions for mass and heat fluxes from the surface of the evaporating droplet:

$$j = \begin{cases} \dfrac{\langle \rho D \rangle}{r_d} \ln\left(\dfrac{1-Y_\infty}{1-Y_l}\right), \\[2ex] M_l p_l \sqrt{\dfrac{\gamma\mu}{RT_l}}, \end{cases}$$

$$j_\tau = j \begin{cases} L + \dfrac{(T_l - T_0)\langle k \rangle \left\langle \dfrac{C_p}{k} \right\rangle}{(e^{j r_d \left\langle \frac{C_p}{k} \right\rangle} - 1)Pe}, & T_d < T_*, \\[3ex] L + h_l - h_s + \dfrac{u_l^2}{2}, & T_d \geqslant T_*. \end{cases} \tag{1}$$
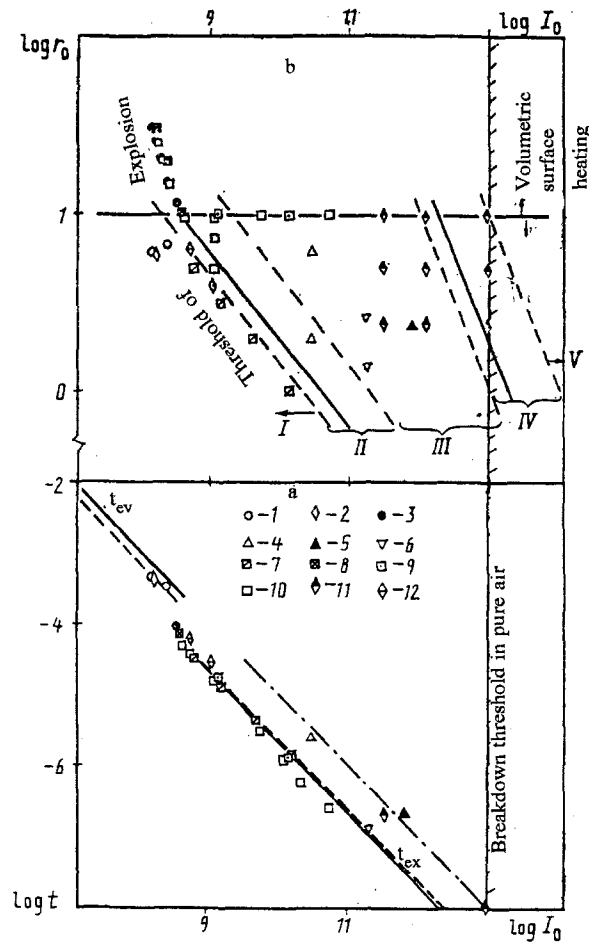
Fig. 1. The evaporation t_{ev} and explosion t_{ex} times as functions of the heat release rate q in a logarithmic scale: dashed line, approximate relations t_{ev} $= 3 \ln 2\rho_w L/q_0$ and $t_{ex} = (h_{ex} - h_0)^2 \rho_{cr}^2/q\langle\rho\rangle$; dot-and-dash line, radiation pulse with the shape in time $I(t) = \frac{2}{5} I_0 \begin{cases} t/t_0, & t < t_0, \\ e^{-(t/t_0-1)/2}, & t \geqslant t_0 \end{cases}$ (a); the map of heating and explosion regimes on the plane of initial droplet r_0 vs heat release rate q in a logarithmic scale (b): 1) results of study [4]; 2) [5]; 3) [7]; 4) [8]; 5) [9]; 6) [10]; 7) [11]; 8) [12]; 9) [13]; 10) [14]; 11) [15]; 12) [16]. t, sec; r_0, μm; I_0, W/m^2.

Here, $\langle\rho D\rangle$, $\langle k\rangle$, and $\langle C_p/k\rangle$ are the temperature-mean quantities; r_d is the variable droplet radius; Y_∞ and Y_l are the mass concentrations of the vapor at infinity and at the outer boundary of the Knudsen layer; T_l, p_l, h_l, and u_l are the temperature, pressure, enthalpy, and velocity of the vapor at the outer boundary of the Knudsen layer; γ is the specific heat ratio for the vapor; μ is the molar mass of the vapor; L is the latent evaporation heat; R is the universal gas constant; and h_s is the enthalpy of the saturated vapor at a temperature of the droplet surface T_d. Implicit relations (1) connect the mass j and heat j_T fluxes to the surface temperature T_d. In laboratory conditions, $T_0 = 288$ K and $p_\infty = 1$ atm. When $T_d < 330$ K, by virtue of linearization, relation (1) becomes explicit (diffusional regime 1). With diffusional-convective regime 2, an iteration process must be organized for the quantities p_l, M_l, and Y_l [3]. Regimes 2 and 3 should be conjugated from the condition of equality of the fluxes calculated by upper and lower formulae (1). The temperature T_*, dividing diffusional regimes 1 and 2 and purely convective regimes 3 and 4, constitutes 381 and 275 K at $r_0 = 1$ and 10 μm, respectively. The sonic regime sets in when $T_d \geq 397$ K. The vapor velocity cannot exceed the speed of sound in the Knudsen layer, since the Knudsen layer is markedly smaller than the droplet radius ($l \ll r_0$), and the flow therein is one-dimensional. Nonetheless, the vapor velocity u_v will rise together with the speed of sound as the heat release rate q increases.

The exact solution of the balance equations of mass and energy on the droplet surface under the assumption that the temperature inside droplet is uniform yields a linear relation between the evaporation time t_{ev} (for example, this is the time of a reduction in the droplet radius by half, and in the mass by an order of magnitude) and the reciprocal of the heat release rate, in accordance with the approximate expression $t_{ev} = 3 \ln 2 \, \rho_w L/q_0$ (where $q_0 = \alpha_0 I_0$) for a quasi-steady evaporation $(dT_d/dt \approx 0)$. This solution is presented by a solid line in the left-hand upper corner (see Fig. 1a), and the approximate solution is shown by a dashed line.* The same figure gives experimental values (circles) and a prediction (rhomb) borrowed from studies [4] and [5], respectively.

Nondimensionalized equations inside the droplet contain two similarity parameters, viz., the water Peclet number $Pe_w = \rho_{cr}h_{cr}r_0 u_w/(k_w T_{cr})$ and the Euler number $Eu = p_{cr}/\rho_{cr}u^2_w$ (where k_w is the characteristic thermal conductivity of the water, and $u_w \lesssim u_0 = r_0/t_0$ and $t_0 = \rho_{cr}h_{cr}/q$ are the characteristic water velocity and heating time), which define heating and explosion regimes for the droplet. The viscosity inside and outside the droplet may be disregarded, since it gives out-of-order terms. By equating Pe_w and Eu to unity, we find levels of the characteristic water velocities $u_T = k_w T_{cr}/\rho_{cr}h_{cr}r_0 = 0.507-0.0507$ m/sec (at $r_0 = 1-10$ μm and $k_w = 0.522$ W/m^3), $u_p = (p_{cr}/\rho_{cr})^{1/2} = 264$ m/sec, and the corresponding levels of the heat release rate $q_T = k_w T_{cr}/r^2_0 = 3.38 \times 10^{14}-3.38 \times 10^{12}$ W/m^3 and $q_p = \rho_{cr}h_{cr}u_p/r_0 = 1.75 \times 10^{17}-1.75 \times 10^{16}$ W/m^3. From the hydrodynamics standpoint, the following heating and explosion regimes exist inside the droplet in order of increasing quantity q [6]: I) heat-conductive, $Pe_w \ll 1$, $q \ll q_T$, and $u_w \ll u_T$; II) heat-conductive-convective, $Pe_w \sim 1$, $q \sim q_T$, $u_w \sim u_T$, and $t_0 \sim t_T = r_0/u_T$; III) convective, $Pe_w \gg 1$, $Eu \gg 1$, and $q_T \ll q \ll q_p$; IV) non-isobaric, $Eu \sim 1$, $q \sim q_p$, $u_w \sim u_p$, and $t_0 \sim t_p = r_0/u_p$; and V) isochoric, $Eu \ll 1$, $q > q_a = \rho_{cr}h_{cr}a/r_0 \sim 10^{18}-10^{17}$ W/m^3, where a is the speed of sound in the water. In heating regime I, the convection (the thermal expansion) of the water may be ignored; in regime II, the convection becomes comparable with the heat conduction, and a threshold of the explosion (of the catastrophic expansion) occurs; in regime III, both the heat conduction and the pressure gradient may be neglected; in regime IV, the pressure gradient becomes appreciable, and the heating time is of the same order as the time of a rapid scattering (of explosion) of the substance; in regime V, the heating is accomplished over a time so short that the water density remains constant, whereas the explosion (scattering) time of the substance is longer than the heating time.

It has been established that with a heat-conductive regime of heating I, the evaporation from the surface proceeds in diffusional regime 1; with regime II, predominantly in regimes 2 and 3; with regime III, chiefly in regimes 3 and 4; and with regimes IV and V, in sonic regime 4.

The convective regime of heating III is the simplest mathematically, since there are no temperature and pressure gradients inside the droplet [6]. On the map of heating regimes r_0-q (or r_0-I_0), constructed in Fig. 1b, this regime occupies the same space as neighboring regimes II and IV taken together. Here, solid lines show the relations $q_T(r_0)$ and $q_p(r_0)$ which are the "centers" of regimes II and IV; dashed lines indicate the boundaries of regimes II and IV offset from them by a "half-order": $q_{\pm 0.5} = 0.5 \times 10^{\pm 1} q_{T,p}$; map symbols (squares, rhombs, etc.) denote experimental [7-10] and theoretical [11-16] studies, for which the times of heating up to the explosion temperature t_{ex} are derived explicitly or can be evaluated (see Fig. 1a).

In regime III, the problem is reduced to solving the system of ordinary differential equations for the temperature inside the droplet, for variable radius, and for velocity, the equation of state for the water [17] and the implicit algebraic relation for the surface temperature $(T - T_d \approx 200$ K has been calculated near regime II). The exact solution, with allowance for temperature variations in thermophysical properties of the water, yields the linear relation $t_{ex} \sim q^{-1}$ both for continuous and pulse radiation (see Fig. 1a). This result is consistent with those obtained by other authors, moreover, in regimes II and IV as well. In the basic approximation, the collapse (evaporation or explosion) time of the optically homogeneous droplet is independent of the radius r_0 and inversely proportional to the heat release rate. The linearized solution $t_{ex} = (h_{ex} - h_0)\rho^2_{cr}/q\langle\rho\rangle$ is also close to exact.

The calculations, carried out in regime II with the water velocity and thermal conductivity taken into account reveal that the velocity becomes inconsiderable as the explosion threshold is approached (from above), whereas near regime IV it attains r_0/t_0.

The explosion threshold is inversely proportional to r_0^2 ($q_{thr} \sim r_0^{-2}$) and practically coincides with the lower boundary of regime II (see Fig. 1b).

No consideration has been given in the study to the processes of plasma formation.

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*$q_0 = 8 \cdot 10^4 I_0$ W/m^3, $q = 8.09 \cdot 10^3 I_0$ W/m^3 at $\alpha_0 = 8 \cdot 10^4$ m^{-1}.

The performed hydrodynamic analysis of the evaporation and explosion may be extended to the case of larger optically inhomogeneous droplets.

NOTATION

r_0, initial droplet radius; I_0, incident radiation intensity; q, heat release rate; ρ, gas or liquid density; T, temperature; p, pressure; h, enthalpy; Kn, Re, Pe, Pr, M, Eu, Knudsen, Reynolds, Peclet, Prandtl, Mach, and Euler numbers; t, time; Y, vapor mass concentration; j, vapor mass flux; j_T, energy flux.

REFERENCES

1. V. E. Zuev, A. A. Zemlyanov, Yu. D. Kopytin, and A. V. Kuzikovskii. High-Power Laser Radiation in an Atmospheric Aerosol [in Russian], Novosibirsk (1984).
2. A. N. Kucherov, VIII All-Union Conference on the Interaction of Optical Radiation with a Substance, Extended Abstracts, Leningrad (1990), Vol. 1, pp. 222-223.
3. A. N. Kucherov, Teplofiz. Vysok. Temp., 29, No. 1, 144-152 (1991).
4. V. Ya. Korovin and E. V. Ivanov, Inzh.-fiz. Zh., 34, No. 5, 807-812 (1978).
5. V. K. Pustovalov and G. S. Romanov, Int. J. Heat Mass Transfer, 28, No. 1, 277-289 (1985).
6. A. I. Kucherov, Inzh.-fiz. Zh., 61, No. 2, 253-261 (1991).
7. V. Ya. Korovin and E. V. Ivanov, in: Abstracts of Reports at 3rd All-Union Symp. on Propagation of Laser Radiation in the Atmosphere, Part I, Tomsk (1975), pp. 93-94.
8. V. A. Bel'ts, A. F. Dobrovolskii, V. P. Nikolaev, and S. S. Khmelevtsov, in: Abstracts of Reports at 4th All-Union Symp. on Propagation of Laser Radiation in Atmosphere, Part 3, Tomsk (1977).
9. V. I. Bukatyi and M. F. Nebol'sin, Abstracts of Reports at 4th All-Union Symp. on Propagation of Laser Radiation in Atmosphere, Part 3, Tomsk (1977), pp. 22-26.
10. Yu. É. Geints, A. A. Zemlyanov, V. A. Pogodaev, and A. E. Rozhdestvenskii, Optika Atmos., 1, No. 3, 27-34 (1988).
11. Yu. N. Grachev and G. M. Strelkov, Kvantovaya Élektron., 3, No. 3, 621-625 (1976).
12. M. N. Gordin, Yu. N. Grachev, V. S. Loskutov, et al., Izv. Akad. Nauk SSSR, Ser. Fiz., 49, No. 3, 450-458 (1985).
13. A. P. Prishivalko, Zh. Prikl. Spektrosk., 33, Issue 2, 351-355 (1980).
14. A. P. Prishivalko, Optical and Thermal Fields inside Light-Scattering Particles [in Russian], Minsk (1983).
15. A. A. Zemlyanov and A. V. Kuzikovskii, Kvantovaya Élektron., 7, No. 7, 1523-1530 (1980).
16. A. V. Butkovskii, Abstracts of Reports at 8th All-Union Conf. on Interaction of Optical Radiation with Substance, Vol. 1, Leningrad (1990), pp. 218-219.
17. J. Himpan, Monatsh. Chem., 86, No. 2, 256-268 (1955).